1. $$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$.

(2)

(b) Find the constants a, b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root.

(2)

(d) Write down the real root of the equation $f(x - 5) = 0$

(1)

a) $x + 3 = 0 \quad \therefore x = -3$

Substitute $x = -3$ into $f(x)$

$f(-3) = 2(-3)^3 + 5(-3)^2 + 2(-3) + 15$

$= -54 + 45 - 6 + 15$ (1)

$f(-3) = 0$

$\therefore (x+3)$ is a factor of $f(x)$ since $f(-3) = 0$ (1)

b) $(x+3)(ax^2 + bx + c) \equiv 2x^3 + 5x^2 + 2x + 15$

$x^3 : a = 2$

$x^2 : 3a + b = 5$

$3(2) + b = 5 \quad \therefore b = -1$ (1)

constant : $3c = 15 \quad \therefore c = 5$ (1)

$\therefore f(x) = (x+3)(2x^2 - x + 5)$

c) $f(x) = 0$: $(x+3)(2x^2 - x + 5) = 0$

if $b^2 - 4ac > 0$, 2 real roots
$b^2 - 4ac = 0$, 1 real root
$b^2 - 4ac < 0$, no real root

$x + 3 = 0$

$x = -3$

(only solution)

$b^2 - 4ac = (-1)^2 - 4(2)(5) = -39 < 0$ (1) (1)

$2x^2 - x + 5 = 0$ has no real solutions

d) $f(x) \longrightarrow f(x-5)$ is a translation $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$f(x-5) = 0$: $-3 + 5 = 2$

only root from (c)

$\therefore$ $x = 2$ is only real solution to $f(x-5) = 0$
(1)

2. $$f(x) = ax^3 + 10x^2 - 3ax - 4$$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a.

You must make your method clear.

(3)

If $(x-1)$ is a factor of $f(x)$, $f(1) = 0$

$f(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4 = 0$ (1)

$a + 10 - 3a - 4 = 0$

$-2a + 6 = 0$ (1)

$2a = 6$

$a = 3$ (1)

3. $\mathrm{f}(x) = (x - 4)(x^2 - 3x + k) - 42$ where k is a constant

Given that $(x + 2)$ is a factor of $\mathrm{f}(x)$, find the value of k.

(3)

$f(-2) = 0$ 1 ← $(x+2)$ is a factor of $f(x)$

$(-2-4)((-2)^2 - 3(-2) + k) - 42 = 0$

$-6(4 + 6 + k) = 42$

$-6(10 + k) = 42$ 1

$-60 - 6k = 42$

$-6k = 102$

$k = -17$ 1

4.

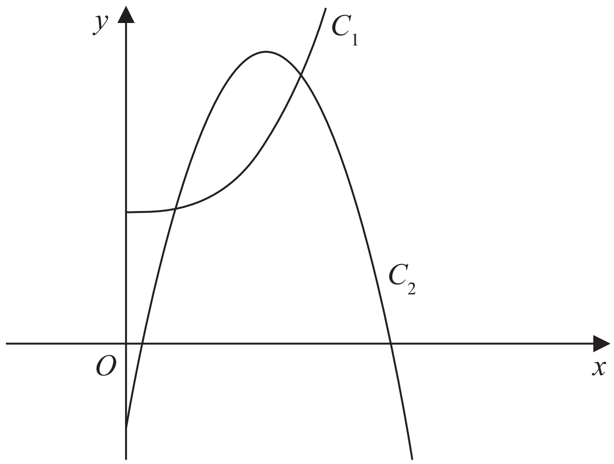

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \frac{1}{2}$

(2)

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

(5)

(a) when $x = \frac{1}{2}$:

C_1: $y = 2(\frac{1}{2})^3 + 10$

$= \frac{41}{4}$

C_2: $y = 42(\frac{1}{2}) - 15(\frac{1}{2})^2 - 7$ 1

$= \frac{41}{4}$

$\therefore$ C_1 and C_2 intersect at $(\frac{1}{2}, \frac{41}{4})$ 1

(b) $2x^3 + 10 = 42x - 15x^2 - 7$ 1

$2x^3 + 15x^2 - 42x + 17 = 0$

$2x-1$ is a factor of this equation – this could be deduced by inspection, trial-and-error or any other valid method.

$$
\begin{array}{r|l}
 & x^2 + 8x - 17 \\
\hline
2x-1 & 2x^3 + 15x^2 - 42x + 17 \\
 & 2x^3 - x^2 \\
\hline
 & 0 + 16x^2 \\
 & 16x^2 - 8x \\
\hline
 & 0 - 34x \\
 & -34x + 17 \\
\hline
 & 0 + 0
\end{array}
$$

(1)

← you don't have to do long division - inspection or other valid algebraic methods are accepted.

$2x^3 + 15x^2 - 42x + 17 = 0 \Rightarrow (2x-1)(x^2 + 8x - 17) = 0$ (1)

$2x - 1 = 0 \Rightarrow x_1 = \frac{1}{2}$

solve $x^2 + 8x - 17$ using a calculator, or the quadratic equation.

from $x^2 + 8x - 17$:

$x_2 = -4 + \sqrt{33}$

$x_3 = -4 - \sqrt{33}$ (1) ($x = \frac{1}{2}$ is the other intercept)

The point P is on the positive side of the y-axis, therefore:

$x = -4 + \sqrt{33}$ (1)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \qquad x \in \mathbb{R}$$

where a is a positive constant.

Given $(x - a)$ is a factor of $f(x)$,

(a) show that

$$a(4a^2 + 5a - 6) = 0$$

(2)

(b) Hence

(i) find the value of a

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3$$

(4)

a) If $(x-a)$ is a factor of $f(x)$, $f(a)=0$

$f(a) = 4a^3 + 5a^2 - 10a + 4a = 0$ ①

$4a^3 + 5a^2 - 6a = 0$

$a(4a^2 + 5a - 6) = 0$ ①

b) i) $a = 0$ or $4a^2 + 5a - 6 = 0$ ← to make $f(a) = 0$

$(4a - 3)(a + 2) = 0$

$a = \frac{3}{4}$ or $a = -2$

$a > 0$ so $a = \frac{3}{4}$ ①

set $= 3$ as the question wants $f(x) = 3$

ii) $f(x) = 4x^3 + 5x^2 - 10x + 4\left(\frac{3}{4}\right) = 3$

$4x^3 + 5x^2 - 10x = 0$ ①

$x(4x^2 + 5x - 10) = 0$

$x = 0$ (1) or $4x^2 + 5x - 10 = 0$

$x = \dfrac{-5 \pm \sqrt{185}}{8}$ (1)

(Total for Question 5 is 6 marks)